Precision Analysis of Polynomial Inflation

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Objectives

In this research we aim to:

- apply an algorithm derived from [1] to remove the slow-roll approximated errors of the polynomial inflationary model, and compare the approximated results, with the exact results;
- show that the polynomial inflationary model [2] is a pathological model, to which the slow-roll approximation produces errors $> \mathcal{O}(10\%)$;
- show that one may conduct a cursory examination of the slow-roll approximated quantities, to predict the efficiency of the slow-roll approximation.

Introduction

Cosmological inflationary models are typically analyzed via an approximation named the slow-roll approximation; which utilizes the scalar potential $V(\phi)$ to extract data on various observables. This is done since obtaining the exact results, which is obtained from the Hubble parameter H is an onerous and laborious process for most inflationary models, since it requires the solving of the Hamiltonian - Jacobi equation. Thus, we will circumvent having to solve the former equation by applying an algorithm derived from [1] to obtain the exact results for the polynomial inflationary model.

Research Background

The results obtained in this research are explained by the following equations derived in [1]:

$$\frac{dF}{dn_s} \simeq \frac{1}{2}\left[\frac{-2+d\eta_v/d\epsilon_v}{-3+d\eta_v/d\epsilon_v}\right]; \qquad \frac{dr}{dn_s} \simeq -8+16\frac{dF}{dn_s};$$

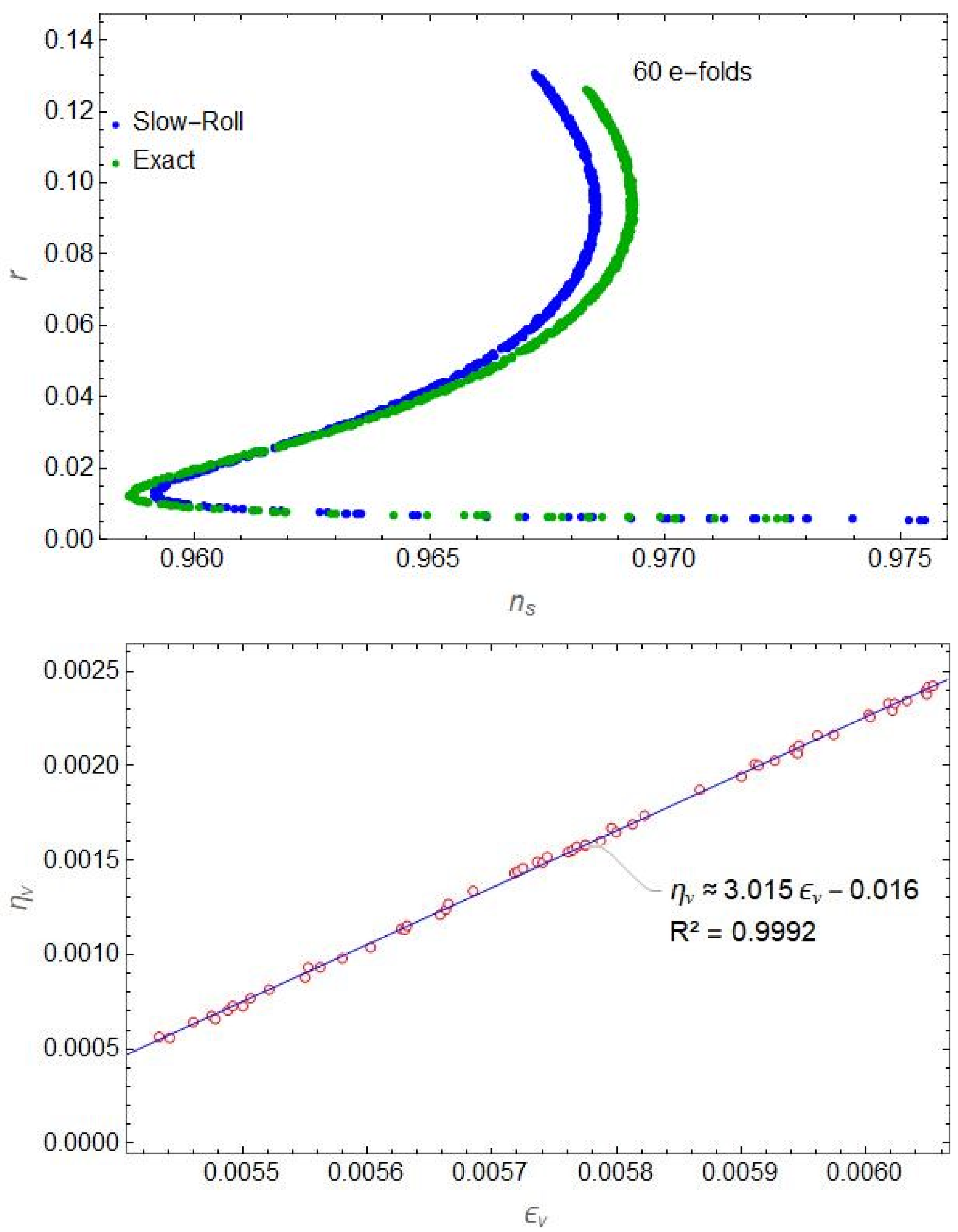
$$\frac{d\epsilon}{d\phi} = \begin{cases} (3-\epsilon)\left(+\sqrt{2\epsilon}-\sqrt{2\epsilon_v}\right) \longleftrightarrow dH/d\phi > 0; \quad dV/d\phi > 0, \\ (3-\epsilon)\left(-\sqrt{2\epsilon}+\sqrt{2\epsilon_v}\right) \longleftrightarrow dH/d\phi < 0; \quad dV/d\phi < 0. \end{cases} \tag{1}$$

where $F \equiv \left(1-\sqrt{\epsilon_v/\epsilon}\right)$, r and n_s are observables, all other quantities with a subscript v are slow-roll approximated, while the converse are exact, and equation (1) is used to obtain the exact results. Since $\frac{dr}{dn_s}$ and $\frac{dF}{dn_s}$ are related linearly to lowest order, we expect to see deviations from the slow-roll approximation in regions where $d\eta_v/d\epsilon_v \approx 3$. Thus, we may just conduct a cursory examination of the slow-roll approximated quantities to predict the efficiency of the slow-roll approximation, before even obtaining the exact results.

Research and Results

To find the deviations between the slow-roll approximation and the exact results, we will first obtain the slow-roll approximated parameters ϵ_v and η_v for the purposes of calculating the approximated version of the observables r and n_s from (2), then numerically solve (1) by inserting ϵ_v to obtain ϵ, and by proxy η to calculate the exact values for the observables mentioned above:

$$V \simeq \frac{1}{2}\phi^2\left(m^2-\sqrt{2}m\lambda sin(\theta)\phi+\frac{\lambda^2}{2}\phi^2\right) \tag{2}$$



(a)

(b)

Figure 1. (a) depicts r vs n_s curves computed via the slow-roll approximation and exactly via (1). Note that the two curves deviate substantially as the slope of the slow-roll curve becomes steep. (b) depicts the η_v vs ϵ_v data near the right-most inflection point in (a) for the slow-roll curve, note that the slope of the fit is ≈ 3.

Research and Discussion

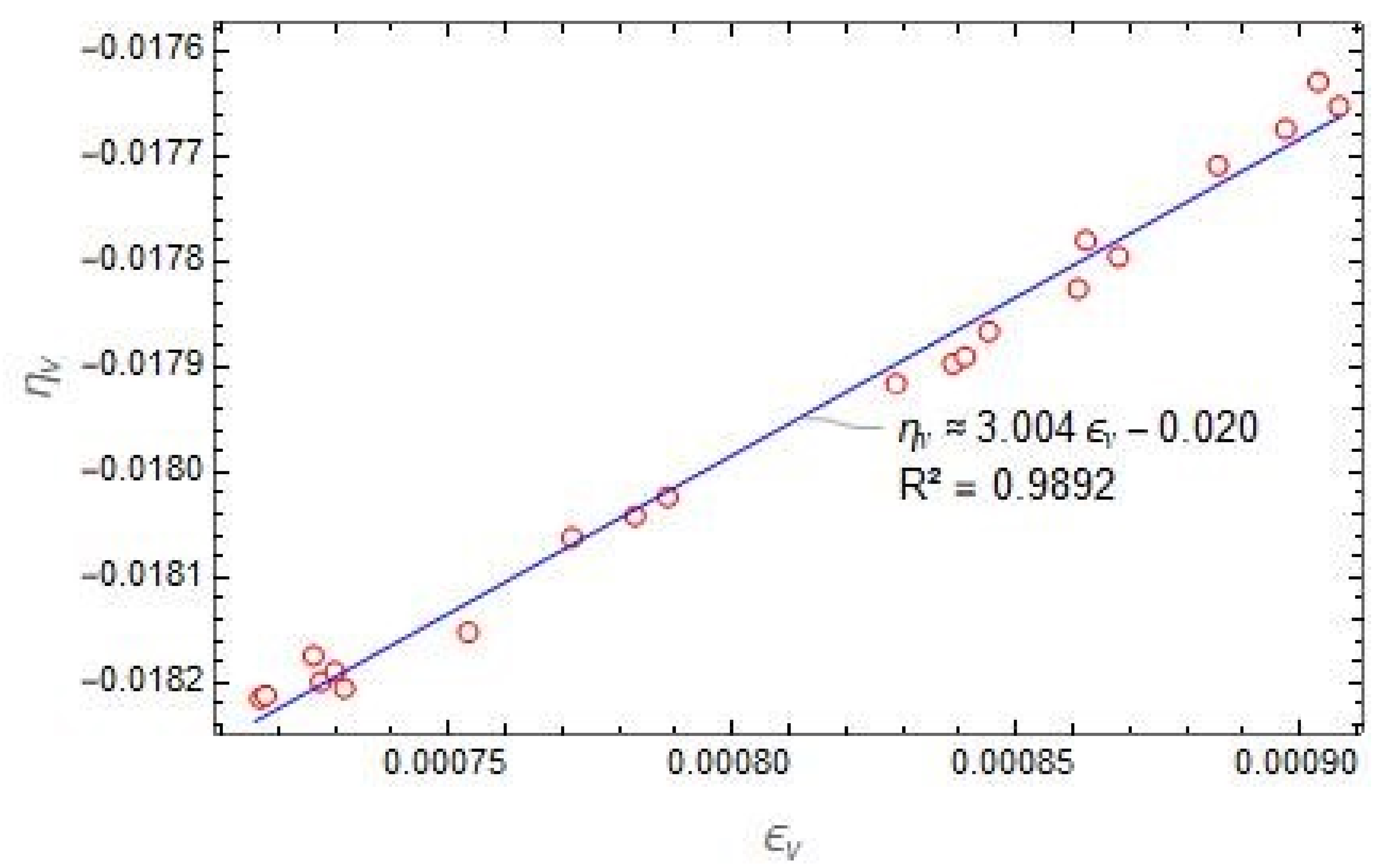


Figure 2. This depicts η_v vs ϵ_v data near the left-most inflection point in (a) for the slow-roll curve. Note that the slope of the fit is ≈ 3.

The slow-roll approximated r vs n_s curve in (a) deviates from it's exact counter part when $\frac{dr}{dn_s}$ grows within an interval of n_s values. Note that the deviation between the two curves isn't as large near the left-most inflection point relative to the converse; which is since $\frac{dr}{dn_s}$ grows over a smaller interval of n_s values. Thus, the proportionality between $\frac{dr}{dn_s}$ and $\frac{dF}{dn_s}$ is apparent in this model.

Research Summary

In this research we have:

- shown that the slow-roll errors may be removed via (1) for the polynomial inflationary potential;
- shown that one may examine the slope of the η_v vs ϵ_v curve to predict the efficiency of the slow-roll approximation;
- shown that the polynomial inflationary model is a pathological model, particularly in regions where $\frac{dr}{dn_s}$ becomes large.

References

[1] M. Civiletti and A. Garifal. Formalizing slow-roll approximation errors: a quantitative and qualitative analysis. *Currently in peer review process via Physical Review D.*

[2] Kazunori Nakayama, Fuminobu Takahashi, and Tsutomu T. Yanagida. Polynomial chaotic inflation in the planck era. *Physics Letters B*, 725(1-3):111–114, Aug 2013.