Objectives

In this research we aim to:

- apply an algorithm derived from [1] to remove the slow-roll approximated errors of the polynomial inflationary model, and compare the approximated results, with the exact results;
- show that the polynomial inflationary model [2] is a pathological model, since it requires the solving of the Hamiltonian-Jacobi equations derived in [1];
- show that one may conduct a cursory examination of the slow-roll approximated quantities, to predict the efficiency of the slow-roll approximation.

Introduction

Cosmological inflationary models are typically analyzed via an approximation named the slow-roll approximation; which utilizes the scalar potential $V(\phi)$ to extract data on various observables. This is done since obtaining the exact results, which is obtained from the Hubble parameter $H$ is an onerous and laborious process for most inflationary models, since it requires the solving of the Hamiltonian-Jacobi equation. Thus, we will circumvent having to solve the former equation by applying an algorithm derived from [1] to obtain the exact results for the polynomial inflationary model.

Research Background

The results obtained in this research are explained by the following equations derived in [1]:

\[
\frac{df}{dn} \approx \frac{-2 + d\eta/\eta_{\epsilon}}{3 + d\eta/\eta_{\epsilon}} \quad \frac{d\eta}{dn} \approx -8 + 6\frac{dF}{dn_{\epsilon}}
\]

\[
\frac{d\eta}{d\phi} \begin{cases} 
(3 - \epsilon) \left[ \frac{\sqrt{2} - \sqrt{3}}{2} \right] \quad & \text{if } dH/d\phi > 0; \\
(3 - \epsilon) \left[ -\frac{\sqrt{2} + \sqrt{3}}{2} \right] \quad & \text{if } dH/d\phi < 0;
\end{cases}
\]

(1)

where $F \equiv \left( 1 - \frac{\sqrt{r^*/\epsilon}}{\epsilon} \right)$, $r$ and $n$ are observables, all other quantities with a subscript $\epsilon$ are slow-roll approximated, while the converse are exact, and equation (1) is used to obtain the exact results. Since $dF/d\epsilon$ and $dn_{\epsilon}$ are related linearly to lowest order, we expect to see deviations from the slow-roll approximation in regions where $d\eta/\eta_{\epsilon} \approx 3$. Thus, we may just conduct a cursory examination of the slow-roll approximated quantities to predict the efficiency of the slow-roll approximation, before even obtaining the exact results.

Research and Results

To find the deviations between the slow-roll approximation and the exact results, we will first obtain the slow-roll approximated parameters $\epsilon$ and $\eta$, for the purposes of calculating the approximated version of the observables $r$ and $n$, from (2), then numerically solve (1) by inserting $\epsilon$, to obtain $\eta$, and by proxy $\eta$, to calculate the exact values for the observables mentioned above:

\[
V \approx \frac{1}{2} m^2 \left( m^2 - \sqrt{2}\sin(\theta)\phi + \frac{\lambda^2}{3} \right)
\]

(2)

The slow-roll approximated $r$ vs $n$, curves computed via the slow-roll approximation and exactly via (1). Note that the two curves deviate substantially as the slope of the slow-roll curve becomes steep. (b) depicts the $\eta$ vs $\epsilon$, data near the right-most inflection point in (a) for the slow-roll curve, note that the slope of the fit is $\approx 3$.

Research and Discussion

In this research we have:

- shown that the slow-roll errors may be removed via (1) for the polynomial inflationary potential;
- shown that one may examine the slope of the $\eta$ vs $\epsilon$, curve to predict the efficiency of the slow-roll approximation;
- shown that the polynomial inflationary model is a pathological model, particularly in regions where $\frac{d\eta}{d\phi}$ becomes large.

References